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Narumi-Katayama and Multiplicative Zagreb Indices of Dutch Windmill Graph

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ABSTRACT - In this paper, we compute Narumi - Katayama index, First multiplicative Zagreb index, Modified multiplicative Zagreb index of Dutch windmill graph.

KEYWORDS - Narumi - Katayama index, First multiplicative Zagreb index, Modified multiplicative Zagreb index of Dutch windmill graph.

1. INTRODUCTION

The Dutch windmill graph is denoted by $D_n^{(m)}$ and it is the graph obtained by taking m copies of the cycle C_n with a vertex in common. The Dutch windmill graph is also called as friendship graph if $n = 3$. i.e., Friendship graph is obtained by taking m copies of the cycle C_3 with a vertex in common. Dutch windmill graph $D_n^{(m)}$ contains $(n-1)m+1$ vertices and mn edges as shown in the figure 1 to 3.

Figure 1 $D_3^{(4)}$

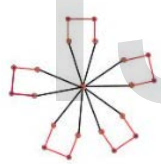

Figure 2 $D_5^{(5)}$

Figure 3 $D_4^{(5)}$

All graphs considered in this paper are finite, connected, loop and without multiple edges. Let $G = (V, E)$ be a graph with n vertices and m edges. The degree of a vertex $u \in V(G)$ is denoted by d_u and is the number of vertices that are adjacent to u. The edge connected the vertices u and v is denoted by uv, Using these terminologies, certain topological indices are defined in the following manner.

Topological indices are numerical parameters of a graph which characterize its topology and are usually graph invariants. Narumi - Katayama index, First and second multiplicative Zagreb index, Modified multiplicative Zagreb index are the degree based molecular descriptor. For further results on $NK(G)$, $\prod_1(G)$, $\prod_1^*(G)$, $\prod_2(G)$ see the papers [4, 5, 6, 7,8] and the references cited there in.

We encourage the reader to consult [1, 2, 3] for basics of Graph Theory, Chemical Graph Theory and molecular descriptors.

Definition 1.1. Let $G = (V, E)$ be a graph and d_u degree of a vertex u then Narumi - Katayama index is defined as $NK(G) = \prod_v d_v(G)$. It was introduced by Narumi and Katayama in [7].

Definition 1.2. Let $G = (V, E)$ be a graph and d_u degree of a vertex u then First multiplicative Zagreb index is defined as $\prod_1(G) = \prod_{v\in E(G)}[d_v(G)]^2$. The First multiplicative Zagreb index was introduced by Guttmann in [8].

Definition 1.3. Let $G = (V, E)$ be a graph and d_u degree of a vertex u then Modified multiplicative Zagreb index index is defined as $\prod_1^*(G) = \prod_{uv\in E(G)}[d_u(G) + d_v(G)]$.

Definition 1.4. Let $G = (V, E)$ be a graph and d_u degree of a vertex u then Second multiplicative Zagreb index is defined as $\prod_2(G) = \prod_{uv\in E(G)}[d_u(G)d_v(G)]$. The second multiplicative Zagreb index was introduced by Guttmann in [8].

2. Main results

Theorem 1. The Narumi - Katayama index of Dutch Windmill graph is $n2^{nm-m+1}$.

Proof. Dutch windmill graph $D_n^{(m)}$ contains $(n-1)m$ vertices of degree two and one vertex of degree 2n. Therefore

$$NK(G) = \prod_v d_v(G)$$
$$= (2n)2^{(n-1)m}$$
$$= n2^{nm-m+1}.$$

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Theorem 2. The First multiplicative index of Dutch Windmill graph is $4^{nm-m+1}n^2$.

Proof .

$$\prod_{v\in v(G)}[d_v\,(G)]^2 = (2n)^2 \times [(2)^2]^{(n-1)m}$$
$$= 4n^2 4^{(n-1)m}$$
$$= 4^{nm-m+1}n^2$$

Theorem 3. The Modified first multiplicative Zagreb index of Dutch Windmill graph is $\left[2^n\left(\frac{m+1}{2}\right)\right]^{2m}$

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Proof: Consider the Dutch windmill graph $D_n^{(m)}$. We partition the edges of $D_n^{(m)}$ into edges of the type $E_{(d_u,d_v)}$ where uv is the edge. In $D_n^{(m)}$ we get edges of the type $E_{(2,2)}$ and $E_{(2m,2)}$. Edges of the type $E_{(2,2)}$ and $E_{(2m,2)}$ are colored in red and black respectively as shown in the figure [4]. The number of these types are given in the table 1.

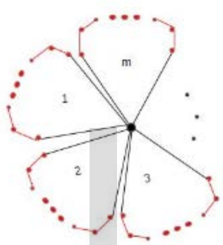

Figure 4
$D_n^{(m)}$

Table 1: Edge partition based on edges of end vertices of each edge.

Edges of the type $E_{(d_u,d_v)}$	Number of Edges
$E_{(2,2)}$	$(n-2)m$
$E_{(2m,2)}$	$2m$

$$\prod\nolimits_1^*(G) = \prod_{uv\in E(G)}[d_u\,(G) + d_v(G)]$$

$$\prod\nolimits_1^*(D_n^m) = \prod_{uv\in E(2,2)}[d_u\,(D_n^m) + d_v(D_n^m)] \prod_{uv\in E(2m,2)}[d_u\,(D_n^m) + d_v(D_n^m)]$$

$$= (2+2)^{|E_{(2,2)}|}\ (2m+2)^{|E_{(2m,2)}|}$$
$$= 4^{(n-2)m}\ (2m+2)^{2m}$$
$$= 4^{(n-2)m}\ (2)^{2m}(m+1)^{2m}$$
$$= 2^{2nm-4m+2m}\ (m+1)^{2m}$$
$$= 2^{2nm-2m}\ (m+1)^{2m}$$
$$= \left[2^n\left(\frac{m+1}{2}\right)\right]^{2m}$$

Theorem 4. The second multiplicative index of Dutch Windmill graph is $4^{nm}m^{2m}$.

Proof . $\prod_2(G) = \prod_{uv\in E(G)}[d_u\,(G)d_v(G)]$

$$\prod\nolimits_2 (D_n^m) = \prod_{uv\in E(2,2)}[d_u\,(D_n^m)d_v(D_n^m)] \prod_{uv\in E(2m,2)}[d_u\,(D_n^m)d_v(D_n^m)]$$
$$= (2\times 2)^{|E_{(2,2)}|}\ (2m\times 2)^{|E_{(2m,2)}|}$$
$$= 4^{(n-2)m}\ (4m)^{2m}$$
$$= 4^{nm-2m+2m}\ m^{2m}$$
$$= 4^{nm}m^{2m}.$$

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